Problem 1

Subject Type

Quantum Mechanics → *Momentum Operator*

$$\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}\psi \Rightarrow p\psi = \hbar k\psi \tag{1}$$

Problem 2

Subject Type

Atomic → *Bragg Diffraction*

Recall the Bragg Diffraction dispersion relation,

$$\lambda = 2d\sin\theta, \tag{2}$$

thus the maximal wavelength λ would be $2d$, choice (D). (One can derive that even if one does not remember the formula. Consider two lattice planes. View them from the side so that they appear as two parallel lines. A wave would hit the both planes at, say, an angle θ from the normal. The wave that reflects off the bottom lattice will have to travel an extra distance, relative to the wave hitting the top plane, equal to $2d\sin\theta$.)

Problem 3

Subject Type

Quantum Mechanics → *Bohr Theory*

Recall the Bohr Equation, $E_n = Z^2/n^2 E_1$, which applies to both the Hydrogen atom and hydrogen-like atoms. One can find the characteristic X-rays from that equation (since energy is related to wavelength and frequency of the X-ray by $E = \lambda f$).

The ratio of energies is thus $E(Z = 6)/E(Z = 12) = 6^2/12^2 = 1/4$, as in choice (A).

Problem 4

Subject Type

Mechanics → *Gravitational Law*

Recall the famous inverse square law determined almost half a millennium ago,

$$F = \frac{k}{r^2}, \tag{3}$$

where $k = GMm$.

The ratio of two inverse-square forces ($r > R$, where R is the radius of the planet or huge heavy object) would be

$$\frac{F(r_1)}{F(r_2)} = \frac{4r_2^2}{r_1^2}. \tag{4}$$

Thus, $\frac{F(R)}{F(2R)} = \frac{4R^2}{R^2} = 4$, which is choice (C).

Problem 5

Subject Type

Mechanics → *Gauss Law*

The inverse-square law doesn't hold inside the Earth, just like how Coulomb's law doesn't hold inside a solid sphere of uniform charge density. In electrostatics, one can use Gauss Law to determine the electric field inside a uniformly charged sphere. The gravitational version of Gauss Law works similarly in this mechanics question since $\nabla \cdot \vec{E} = \rho_e \Rightarrow \nabla \cdot \vec{g} = \rho_M$, where ρ_M is the mass density of M. In short, the gravitational field $\vec{g}$ plays the analogous role here as that of $\vec{E}$ Thus, $\int \vec{g} \cdot d\vec{a} = \int \rho dV$.

So, for $r < R$, $g(4\pi r^2) = \rho \frac{4}{3}\pi r^3 \Rightarrow g = r\frac{\rho}{3}$, where one assumes ρ is constant.

To express the usual inverse-square law in terms of ρ, one can apply the gravitational Gauss Law again for $r > R$, $g(4\pi r^2) = \rho \frac{4}{3}\pi R^3 \Rightarrow g = \frac{R^3}{r^2}\frac{\rho}{3}$.

Since $\vec{F} = m\vec{g}$ Therefore,

$$\frac{F(R)}{F(R/2)} = 2R. \tag{5}$$

Problem 6

Subject Type

Mechanics → *Method of Sections*

By symmetry, one can analyze this problem by considering only *one* triangular wedge. The normal force on one wedge is just $N = (m + M/2)g$, since by symmetry, the wedge (m) carries half the weight of the cube (M). The frictional force is given by $f = \mu N = \mu(m + M/2)g$.

Sum of the forces in the horizontal-direction yields $F_x = 0 \leq f - N_M/\sqrt{2} = \mu(m+M/2)g - Mg/2$ for static equilibrium to remain valid. (Note that the normal force of the cube is given by $N_M = Mg/\sqrt{2}$ since, summing up the forces perpendicular to the plane for M, one has, $N_M \sin(\pi/4) = Mg/2$. Also, note that it acts at a 45 degree angle to the wedge.)

Solving, one has $\mu(m + M/2)g \geq Mg/2 \Rightarrow M \leq \frac{2\mu m}{1-\mu}$.

(In a typical mechanical engineering course, this elegant method by symmetry is called the *method of sections*.)

Problem 7

Subject Type

Mechanics → *Normal Modes*

For normal mode oscillations, there is *always* a symmetric mode where the masses move together as if just one mass.

There are three degrees of freedom in this system, and ETS is nice enough to supply the test-taker with two of them. Since the symmetric mode frequency is not listed, choose choice it!—as in (A).

Problem 8

Subject Type

Mechanics → *Torque*

The problem wants a negative z component for τ. Recall that $r \times F = 0$ whenever r and F are parallel (or antiparallel). Thus, choices (A), (B), (E) are immediately eliminated. One can work out the cross-product to find that (D) yields a positive τ_z, thus (C) must be it.

Problem 9

Subject Type

Electromagnetism → *Current Directions*

The opposite currents cancel each other, and thus the induction (and field) outside is 0.

Problem 10

Subject Type

Electromagnetism → *Image Charges*

The conductor induces image charges -q and -2q since it is grounded at $x = 0$. Since these are (mirror) image charges, each charge induced is the same distance from the conducting plane as its positive component.

The net force on q is just the *magnitude* sum of the positive charge 2q and the two induced charges, $\sum F = \frac{q^2}{4\pi\epsilon_0}\left(1/(a^2) + 2/(2a^2) + 2/(a^2)\right) = \frac{q^2}{4\pi\epsilon_0 a^2}(1 + 1/2 + 2) = \frac{q^2}{4\pi\epsilon_0 a^2}\frac{7}{2}$, as in choice (E).

Problem 11

Subject Type

Electromagnetism → *RC Circuit*

The energy of a capacitor C with voltage V across it is given by $U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$. ($Q = CV$ derives the other variations of the energy.)

From Ohm's Law, one arrives at the relation between charge and time, $Q/C + \dot{Q}R = 0 \Rightarrow \frac{Q}{RC} = -\frac{dQ}{dt} \Rightarrow -\frac{dt}{RC} = \frac{dQ}{Q}$. Integrating both sides, one finds that $Q(t) = Q_0 e^{-t/(RC)}$.

Plugging this into the energy equation above, one has $U \propto Q(t)^2 \propto e^{-2t/(RC)}$. Twice time required for the energy to dissipate by 2 is thus given by $1/2 = e^{-t/(RC)} \Rightarrow t_{1/2} = RCln(2$. Divide it by 2 to get choice (E).

Problem 12

Subject Type

Electromagnetism → *Potential*

A Potential V is related to the electric field E by $\vec{E} = \nabla V$.

Since the problem supplies the approximation tool that the planes are quite large, one can assume the field is approximately constant. The remaining parameter that can't be thrown out by this approximation is the angle, and thus the only choice that yields $\frac{d}{d\phi}V = constant$ is choice (B).

Problem 13

Subject Type

Electromagnetism → *Maxwell's Equations*

Magnetic monopoles remain a likeable (even lovable) theoretical construct because of their ability to perfectly symmetrize Maxwell's equations. Since the curl term has an electric current, the other curl term should have a magnetic current. ($\nabla \cdot B \neq 0$ is taken to be obvious in presence of magnetic charge.) The answer is thus (D), and the revised equations are,

$$\nabla \times H = J_e + \frac{\partial D}{\partial t} \quad (6)$$

$$\nabla \times E = -J_m - \frac{\partial B}{\partial t} \quad (7)$$

$$\nabla \cdot D = \rho_e \quad (8)$$

$$\nabla \cdot B = \rho_m. \quad (9)$$

Problem 14

Subject Type

Statistical Mechanics → *Blackbody Radiation Formula*

Recall

$$P = ut \propto T^4, \tag{10}$$

where P is the power and u the energy and T the temperature.

So, initially, the blackbody radiation emits $P_1 = kT^4$. When its temperature is doubled, it emits $P_2 = k(2T)^4 = 16kT^4$.

Recall that water heats according to $Q = mc\Delta T = \kappa\Delta T$. So, initially, the heat gain in the water is $Q_1 = \kappa(0.5^\circ)$. Finally, $Q_2 = \kappa x$, where x is the unknown change in temperature.

Conservation of energy in each step requires that $kT^4 t = \kappa/2$ and $16kT^4 t = \kappa x$, i.e., that $P_i t = Q_i$. Divide the two to get $\frac{1}{16} = \frac{2}{x} \Rightarrow x = \Delta T = 8^\circ$. Assuming the experiment is repeated from the same initial temperature, this would bring the initial 20° to 28°, as in choice (C).

Problem 15

Subject Type

Statistical Mechanics → *Heat Capacity*

Note that this problem wants the regime of high temperatures, and thus the answer is *not* $\frac{5}{2}R$ from classical thermodynamics, but rather $\frac{7}{2}R$.

The problem suggests that a quantized linear oscillator is used. From the energy relation $\epsilon = \left(j + \frac{1}{2}\right)\hbar\nu$, one can write a partition function and do the usual Stat Mech jig. Since one is probably too lazy to calculate entropy, one can find the specific heat (at constant volume) from $c_v = \frac{\partial U}{\partial T}\big|_v$, where $U = NkT^2\left(\frac{\partial Z}{\partial T}\right)_V$, where N is the number of particles, k is the Boltzmann constant.

There are actually three contributions to the specific heat at constant volume. $c_v = c_{translational} + c_{rotational} + c_{vibrational}$. Chunk out the math and take the limit of high temperature to find that $c_v = \frac{7}{2}R$.

Problem 16

Subject Type

Thermodynamics → *Carnot Engine*

Recall the common-sense definition of the efficiency e of an engine,

$$e = \frac{W_{accomplished}}{Q_{input}}, \tag{11}$$

where one can deduce from the requirements of a Carnot process (i.e., two adiabats and two isotherms), that it simplifies to

$$e = 1 - \frac{T_{low}}{T_{high}} \tag{12}$$

for Carnot engines, i.e., engines of maximum possible efficiency. (Q_{input} is heat put into the system to get stuff going, $W$is work done by the system and T_{low} (T_{high}) is the isotherm of the Carnot cycle at lower (higher) temperature.)

The efficiency of the Carnot engine is thus $e = 1 - \frac{800}{1000} = 0.2$, where one needs to convert the given temperatures to Kelvin units. (As a general rule, most engines have efficiencies lower than this.) The heat input in the system is $Q_{input} = 2000J$, and thus $W_{accomplished} = 400J$, as in choice (A).

Problem 17

Subject Type

Lab Methods → *Oscilloscope*

This problem can be solved by elimination. Since one is given two waves, one with twice the frequency of the other, one can approximate the superposed wave (which shows up on the oscilloscope) as $\sin(\omega t) + \sin(2\omega t)$.

The summed wave no longer looks like a sine wave. Instead, it looks like a series of larger amplitude humps alternating with regions of smaller amplitudes.

However, since one is not supplied with a graphing calculator on the test, one can qualitatively eliminate the other choices based on the equation above. It is obviously not choices (D) and (E) since the superposition is still a one-to-one function. It isn't choice (C) or (B) since those are just sin waves (cosine waves are just off by a phase), and one knows that the superposed wave would look more complicated than that. Thus, one arrives at choice (A), which is a zoomed-in-view of the superposition above.

Problem 18

Subject Type

Lab Methods → *Coax Cable*

Elimination time. The first-pass question to answer is *why is it important that a coax cable be terminated at an end*: (A) Perhaps...

(B) Probably not. Terminating the cable at an end would not help heat dissipation and thus should not prevent overheating.

(C) Perhaps...

(D) Probably not, since termination should attenuate the signal rather than to prevent it.

(E) Probably not, since image currents should be canceled by the outer sheath.

Choices (A) and (C) remain. Now, use the second fact supplied by ETS. The cable should be terminated by its characteristic impedance. Characteristic impedance has to do with resonance. Thus, it should prevent reflection of the signal.

Problem 19

Subject Type

Mechanics → *Mass of Earth*

If one does not remember the mass of the earth to be on the order of $10^2 4kg$, one might remember the mass of the sun to be $10^3 0kg$. Since the earth weighs much less than that, the answer would have to be either (A) or (B). The problem gives the radius of the earth, and one can assume that the density of the earth is a few thousand kg/m^3 and deduce an approximate mass from $m = \rho V$. The answer comes out to about $10^2 2$, which implies that the earth is probably a bit more dense than one's original assumption. In either case, the earth *can't* be, on average, uniformly $10^9 kg/m^3$ dense. Thus (A) is the best (and correct) answer.

Problem 20

Subject Type

Optics → *Missing Fringes*

Missing fringes in a double-slit interference experiment results when diffraction minima cancel interference maxima.

From a bit of phasor analysis, one can derive the diffraction factor $\beta/2 = \pi w/\lambda \sin\theta$ and the interference factor $\delta/2 = \pi d/\lambda \sin\theta$, where w is the width of the slits and d is the separation (taken from slit centers). The angles belong in the intensity equation given by $I \propto \sin(\beta/2)^2 \cos(\delta/2)^2$.

Thus, the condition for a double-slit diffraction minimum is given by $\delta/2 = m_d\pi = \pi w/\lambda \sin\theta \Rightarrow m_d\lambda = w \sin\theta$.

Also, the condition for interference maximum is given by $\beta/2 = m_i\pi = \pi d/\lambda \sin\theta \Rightarrow m_i\lambda = d \sin\theta$.

Now, one needs to find the choice that allows for an integer m_d. This immediately eliminates choices (A) and (B). But, this leaves choices (C), (D), and (E). Among the remaining choices, there is only one choice that allows for slits that are smaller than the separation. This is choice (D). Take it.

Problem 21

Subject Type

Optics → *Thin Film*

Elimination time.

I. Can't be this, since one knows from basic thin-film theory that choice IV is right. (None of the letter choices allow for both choices I and IV.)

II. Thin film theory has $2t = \lambda/2$ for constructive interference and $2t = \lambda$ for destructive interference. Thus, the thickness of the film is smaller than that of the light. (Search on the homepage of this site for more on thin film theory—it is explained in the context of other problems.)

III. This phase change allows for the half-integer constructive interference.

IV. Phase change only occurs when light travels from a medium with lower index of refraction to a medium with higher index of refraction. Since at the back surface, the light would be going from higher to lower index of refraction, there is no phase change.

Thus, choice (E).

Problem 22

Subject Type

Optics → *Telescope*

The magnification for a telescope is related to the focal length for the eyepiece and objective by $M = f_o/f_e$. (Note that it is the eye-piece that magnifies it. The objective merely sends an image that's within view of the eye-piece. However, magnification is inversely related to focal length.)

The problem gives angular magnification to be $M = 10 = f_o/f_e \Rightarrow f_e = f_o/10 = .1m$. The distance between the objective and eyepiece is the sum of the focal lengths (since the light comes from infinity). $d = f_o + f_e = 1.1$m as in choice (D).

Problem 23

Subject Type

Statistical Mechanics → *Fermi Temperature*

(Much of the stuff I classified as Stat Mech might also be considered Condensed Matter or Solid State Physics. They are classified as thus because the Stat Mech book I mentioned in the booklist on the site $http://grephysics.yosunism.com$ is perhaps the best intro to all this.)

The Fermi velocity is related by $\epsilon_F = kT_F = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2kT_F}{m}}$, where ϵ_F is the fermi energy, and T_F is the Fermi temperature.

One should know by heart the following quantities, $k = 1.381E-23$ and $m = 9.11E-31$ (but then again, they are also given in the table of constants included with the exam). Plug these numbers into the expression above to find v,

$v = \sqrt{\frac{2kT_F}{m}} \approx \sqrt{\frac{2\times 1.4E-23\times 8E5}{9E-31}} = \sqrt{\frac{2.8E-23\times 8E5}{9E-31}} \approx \sqrt{0.3E8\times 8E5} = \sqrt{24/10E13} \approx \sqrt{10^{12}} = 10^6$, the choice that comes closest to this order is choice (E).

Note that the hardest part of this problem is the approximation bit. No calculators allowed. Sadness.

Problem 24

Subject Type

Atomic → *Bonding*

Solid Argon is a Nobel gas. It has a full shell of outer electrons, and thus it cannot bond in anything but van der Walls bonding, which isn't really bonding, but more weak like charge-attraction.

One can arrive at this choice by elimination: (A) Ionic bonding occurs when one atom is a positive ion and the other the compensating negative ion. Since solid Argon isn't an ion, it can't do this.

(B) Covalent bonding occurs when electrons are shared between atoms. This only happens when the atom has unfilled orbitals. (Incidentally, it only occurs when two electrons are of opposite spins due to the Pauli Exclusion Principle. That is, they must have different quantum numbers so that they can both remain stable in a low energy state.)

(C) No partial charge-analysis needed.

(D) Argon isn't a metal.

(E) This is the one that remains.

Problem 25

Subject Type

Advanced Topics → *Particle Physics*

Choice (A) and (C) involve atoms, which are quite massive. Choice (B) involves protons, which are also pretty massive.

Problem 26

Subject Type

Lab Methods → *Log-Log graph*

Since initially, the counts per minute is $6E4$, the half-count amount would be $3E4$. This occurs between 5 and 10 minutes. Choice (B) seems a good interpolation.

Problem 27

Subject Type

Quantum Mechanics → *Uncertainty*

This problem looks much more complicated than it actually is. Since k and x are fourier variables, their localization would vary inversely, as in choice (B).

Problem 28

Subject Type

Quantum Mechanics → *Probability*

One doesn't actually need to know much (if anything) about spherical harmonics to solve this problem. One needs only the relation $P = \sum_i |\langle Y_i^3 | \psi(\theta, \phi)\rangle|^2$. Since the problem asks for states where $m = 3$, and it gives the form of spherical harmonics employed as Y_l^m, one can eliminate the third term after the dot-product.

So, the given wave function $\psi(\theta, \psi) = \frac{1}{\sqrt{30}}\left(5Y_4^3 + Y_6^3 - 2Y_6^0\right)$ gets dot-product'ed like $|\langle Y_i^3 | \psi(\theta, \phi)\rangle|^2 \left(\frac{1}{\sqrt{30}}\left(5Y_4^3 + Y_6^3\right)\right.$ $\frac{25+1}{30} = \frac{13}{15}$, as in choice (E).

Problem 29

Subject Type

Quantum Mechanics → *Bound State*

Tunneling should show exponential decay for a finite-potential well, and thus choice (E) is eliminated. Choice (C) is eliminated because the wave function is not continuous. One eliminates choice (D) because the bound-state wave functions of a finite well isn't linear. The wave function for a bound state should look similar to that of an infinite potential well, except because of tunneling, the well appears larger—thus the energy levels should be lower and the wave functions should look more spread out. Choice (B) shows a more-spread-out version of a wave function from the infinite potential well.

Problem 30

Subject Type

Quantum Mechanics → *Bohr Theory*

The ground state binding energy of positronium is half of that of Hydrogen. This is so because the energy is proportional to the reduced mass, and that of the positronium has a reduced mass of half that of Hydrogen.

Thus, from the Bohr formula, one has $E = Z^2 E_1/n^2$, where $E_1 = E_0/2$ and E_0 is the ground state energy fo Hydrogen.

Since $Z = 2$, then for $n = 2$, the energy is $E_1/4 = 3_0/8$, as in choice (E).

Problem 31

Subject Type

Atomic → *Spectroscopic Notations*

Spectroscopic notation is given by ${}^{2s+1}L_j$, and it's actually quite useful when one is dealing with multiple particles. $L \in (S, P, D, F)$, respectively, for orbital angular momentum values of $0, 1, 2, 3$. $s = 1/2$ for electrons. j is the total angular momentum.

Knowing the convention, one can plug in numbers to solve $3 = 2s + 1 \Rightarrow s = 1$. Since the main-script is a S, $l = 0$. The total angular momentum is $j = s + l = 1$.

Problem 32

Subject Type

Electromagnetism → *Circuits*

Power is related to current and resistance by $P = I^2 R$. The resistor that has the most current would be R_1 and R_{eq} (the equivalent resistance of all the resistors except for R_1), since all the other resistors share a current that is split from the main current running from the battery to R_1. Since $R_{eq} < R_1$, the most power is thus dissipated through R_1, as in choice (A).

Problem 33

Subject Type

Electromagnetism → *Circuits*

One can find the voltage across R_4 quite easily. The net resistance of all resistors except R_1 is $R_{eq} = ((1/R_3 + 1/R_4)^{-1} + R^5)^{-1} + 1/R_2 = 25\Omega$. Kirchhoff's Loop Law then gives $V = I(R_1 + R_{eq}) \Rightarrow I = 3/75A$.

Now that one knows the current, one trivially finds the voltage across R_2 to be $IR_2 = 1$ V. $I^{'}(R_{34} + R_5) = 1$, since the resistors are in parallel.

Since $R_{34} = 1/R_3 + 1/R_4 = 1/60 + 1/30 = 20\Omega$, the current $I^{'} = 1/(R_{34} + R_5) = 1/50$. The voltage across either R_3 or R_4 is just $1 - I^{'} R_5 = 1 - 30/50 = 0.4$, as in choice (A).

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